Quantum Computing Bootcamp Assignment-2

**Q1. Foundations of Classical Mechanics**

1. **State Newton’s fundamental equation of motion**

Ans: Newton's laws of motion are three classical physics laws that define how an item moves and what forces operate on it.

* A body in motion remains in motion, and a body at rest remains at rest, unless acted on by a force.
* F = m\*a is the equation for force, which is defined as mass times acceleration. Alternatively, the rate of change of a body's momentum equals the force exerted on it: F = Δp/Δt.
* Every action has an equal and opposite reaction.

1. **Using F = −kx, derive the equation of motion for a harmonic oscillator**

Ans: Hooke's Law describes the restoring force F of a harmonic oscillator as follows:

*F= - kx*

Using Newton's second law:

*F = ma*

*a = d2x/dt2*

*ma = - kx*

*m d2x/dt2 = -kx*

*d2x/dt2 + (k/m)x=0*

*We know that Ω2 = (k/m)*

*d2x/dt2 + Ω2 x=0*

This is the typical equation of motion for a basic harmonic oscillator.

1. **Write the general solution and explain its physical significance**

Ans: The general solution to the differential equation:

*d2x/dt2 + Ω2 x=0*

Its solution is;

*x(t)=Acos(ωt) + Bsin(ωt)*

*=Ccos(ωt+ϕ)*

Where,

* C is the amplitude of oscillation
* ω is the angular frequency
* ϕ is the phase constant (determined by initial conditions)

This solution depicts oscillatory motion, in which the system experiences periodic displacement. The frequency ω controls the rate of oscillation, while A and B specify the beginning location and velocity. This equation describes vibrations in mechanical systems, wave phenomena, and even quantum activity in atomic structures.

**Q2. Advanced Formulations in Mechanics**

1. **Write the Euler-Lagrange equation**

Ans: The Euler-Lagrange equation is a fundamental equation in Lagrangian mechanics, based on the concept of least action. It states that if a functional J[y]= ∫F(x,y,y1) dx has an extremum at y(x), then the function y(x) must satisfy the equation:

where:

* F(x,y,y1) is the **Lagrangian function** (depending on y and its derivative y1).
* The equation provides necessary conditions for extremals of a functional **J[y]**, often appearing in variational problems.

1. **State Hamilton’s equations:**

Ans: A collection of first-order differential equations known as Hamilton's equations is provided by:

Where H is the **Hamiltonian**, typically H = T + V (total energy), q​ are generalized coordinates, and p are the **generalized momentam.**

1. **Explain their significance in both classical and quantum mechanics**

Ans: Compared to Newtonian physics, the Euler-Lagrange and Hamiltonian formulations of classical mechanics give a more comprehensive framework and deeper insights into motion and conservation principles. Phase-space dynamics is introduced by Hamilton's equations, which simplify difficult issues.

The Hamiltonian in quantum mechanics drives system development through the Schrödinger equation, which connects classical Poisson brackets to quantum commutators. These theories unified classical and quantum mechanics, influencing modern physics such as quantum field theory and general relativity.

**Q3. Linear Algebra Fundamentals**

* **Define: vector space, basis, linear independence**

Ans: A space comprised of vectors, collectively with the associative and commutative law of addition of vectors and also the associative and distributive process of multiplication of vectors by scalars is called vector space.

A collection of linearly independent vectors that span the entire vector space V is referred to as a basis for vector space V.

In a vector space, a set of vectors is said to be linearly independent if no vector in the set can be expressed as a linear combination of the other vectors in the set. A set of vectors {v1, v2, . . . , vn} is linearly independent if the equation:

*c1v1 + c2v2 + . . . + cnvn = 0*

has only the trivial solution c1 = c2 = . . . = cn = 0.

In contrast, if there exist non-zero scalars c1, c2 . . . cn such that the equation above holds, then the set of vectors is linearly dependent.

* **What are inner products and outer products. Define them with proper bra-ket notation with examples.**

Ans: The wave function is written as |Ψ⟩ and referred to as a ket vector. The complex conjugate Ψ∗=⟨Ψ| is a bra vector, where ⟨aΨ|=a∗⟨Ψ|. The product of a bra and ket vector, ⟨α∣β⟩ is therefore an inner product (scalar), whereas the product of a ket and bra |β⟩⟨α| is an outer product (matrix).

**Inner product example:**

**Outer product example:**

* **Write the computational basis states for a 2-qubit system using Dirac notation**

Ans: To describe this combined system, we will need a way to combine their corresponding vector spaces. This is done with the tensor product.

**Q4. Operators in Quantum Mechanics**

* **Define Hermitian and unitary operators with examples**

Ans: An operator A is said to be normal if

AA†=A†A

An operator is diagonalisable if, and only if, it is normal.

A is said to be Hermitian if A = A†

A normal operator is Hermitian if, and only if, it has real eigenvalues.

A linear operator A is unitary if

Unitary operators are normal and therefore diagonalisable. Unitary operators are norm-preserving and invertible. All eigenvalues of a unitary operator have modulus 1.

* **State the conditions for unitarity and hermiticity**

Ans:

**Conditions for unitarity:**

**Conditions for hermiticity:**

* **Write Pauli matrices and their eigenvalues**

Ans: The Pauli matrices are a set of three 2×2 Hermitian and unitary matrices used in quantum mechanics to describe spin operators. They are defined as:

The eigenvectors of a matrix M for a given eigenvalue λ, we want to find a basis for the null space of M−λI.

In this case, as each M is 2×2 and we have two eigenvalues, the dimension of each eigenspace is 1 and we are looking for one eigenvector for each eigenvalue.

For example, for M=σzand λ=1,

σ3−1.I=

and the eigenvector , as this is a basis vector for the null space. Each Pauli matrix has eigenvalues +1 and -1

**Q5. Quantum Harmonic Oscillator**

1. **Write the Hamiltonian operator for harmonic oscillator**

Ans:

* = Hamiltonian operator (total energy operator)
* = Momentum operator:
* = Position operator
* m = Mass of the particle
* ω = Angular frequency of the oscillator
* ℏ = Reduced Planck’s constant

1. **Explain how to find energy eigenvalues**

Ans: We solve the Schrödinger equation to get the energy eigenvalues in quantum mechanics, especially for a system controlled by a Hamiltonian :

Where,

* = Hamiltonian operator (total energy operator)
* ∣ψ⟩ is the eigenstate (wavefunction)
* E is the **energy eigenvalue** (the quantized energy level)

1. **State the significance of ladder operators**

Ans: Ladder operators, also known as creation and annihilation operators, are powerful tools that make solving the quantum harmonic oscillator problem much easier and more elegant. Instead of directly solving differential equations, you can use these operators to move between energy states smoothly.

**References:**

* [Quantum Physics II, Lecture Notes 4](https://ocw.mit.edu/courses/8-05-quantum-physics-ii-fall-2013/4de6d044fa9d7e5b8998c5f8ca984a42_MIT8_05F13_Chap_04.pdf)
* [quantum-states.pdf](https://ak2316.user.srcf.net/files/handouts/quantum-states/quantum-states.pdf)
* [QC\_2017.pdf](https://www.cl.cam.ac.uk/teaching/1718/QuantComp/QC_2017.pdf)